Learning Lessons

The Research Publication of King Edward VI Grammar School, Chelmsford

Learning, assessment & the use of problem solving in Maths

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"The methods used by different individuals should be open for examination, and discussion, and the goal of all participants should be to search for better methods" (Carpenter,T., Fennema,E., Fuson, K., Hiebert, J., Human, P., Murray,H. et al. 1997 p.39)

Following on from her publication a year ago on perceptions of Mathematics, Rachael here presents her vision of how learning and assessment in Maths need to be embedded in deep learning and the use of problem solving.

Introduction and a personal view on learning and assessment

Within this assignment I shall attempt to put forward my views on curricula, learning and assessment (and how they affect my teaching), and compare them with a number of learning theories. I will then describe some curriculum development that I am undertaking as Head of Department at KEGS. The resources and ideals that I describe will be analysed and justified in terms of pupils' learning, assessment, and the first section of the assignment.

As a teacher the reasoning behind everything I do within my school (and beyond) is that pupils should enjoy learning and they should gain the tools to learn successfully. They should want to continue their learning beyond the classroom and into adulthood (not necessarily in an institution). I feel that the way they will do this is through understanding how and why they are learning. Effective learning is achieved by making links, discovering new ideas yourself, building upon previous knowledge, adapting current frameworks and asking questions about what goes next. Students need help "to build structures that are more complex, powerful and abstract" (Cobb, p. 89) but it is important that they do this without "teaching by imposition…insisting that they use prescribed methods" (Cobb, 1988 p. 96). This is exemplified by the study of numbers. Pupils early in their primary careers learn about positive whole numbers, these are developed by the addition of fractions, decimals and negatives. Later in their secondary schooling pupils' models are extended to include irrational numbers and eventually in some cases complex numbers. Each time a new concept is met pupils' own maps of mathematics are adapted; branches are added and understanding refined.

This constructivist view, in line with Piaget and Vygotsky, strongly affects the way I teach; lessons contain a large amount of self-discovery and plans will often change to suit student queries. Although at times I do feel the tension "between supporting the initiative and problem-solving abilities of students and...promoting the construction of mathematically important concepts and skills" (Carpenter,T.,

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"Developing the capacity to pursue new and interesting ideas with fellow educational practitioners to have a real impact on the lives and life chances of young people"

Volume 2 issue 7 February 2010

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Editor: T Carter

Fennema,E., Fuson, K., Hiebert, J., Human, P., Murray,H. et al. 1997 p.29) this is primarily due to feeling forced to "neglect what is educationally in the best interests of their (my) pupils in favour of work directly specifically to the examination" (Howson, 1993, p. 55). Working on the curriculum development below has reinforced in my own thoughts that this isn't necessarily an issue we have to face - that it is possible to encourage problem solving skills and initiative whilst also ensuring pupils construct the mathematical models required. Students not only need to see the relevance of the mathematics they are learning to their lives but also gain a love of the subject for its own sake; it is beautiful and elegant. I don't hold with the instrumental idea that I am just "serving an extrinsic aim or external purpose such as producing citizens who will benefit society" (Mill and Soler, p. 58) However, I recognise the fact that Mathematics is an "essential prerequisite for technological and scientific advancement" (Howsen, p. 54). Without mathematical understanding scientists, economists, computer developers and others cannot make progress in their area of study. The setting in which students attempt mathematical activity is also incredibly important. I strongly feel that a silent classroom is not necessarily a good one where learning is being achieved. Pupils need to have a use for their knowledge; they need to communicate and collaborate with others.

The idea of social or situational learning comes through strongly in my teaching. A typical lesson will include a large amount of discussion (not necessarily teacher led). Pupils will be given time to talk through ideas with partners or groups before presenting solutions, silence whilst working on a problem is rare. Ideas and solutions are challenged by all, including pupils challenging both the text book and myself. "The methods used by different individuals should be open for examination, and discussion, and the goal of all participants should be to search for better methods" (Carpenter,T., Fennema,E., Fuson, K., Hiebert, J., Human, P., Murray,H. et al. 1997 p.39). Pupils are expected to give full explanations and to use questioning to clarify any areas of which they are unclear. "Engaging in open, honest, public discussions...is the best way to gain deeper understanding of the subject" (Carpenter,T., Fennema,E., Fuson, K., Hiebert, J., Human, P., Murray,H. et al. 1997 p. 39).

Pupils are given a mixed diet of resources. Problem solving is a strong component. I use puzzles, games and students' own written questions alongside a small amount of bookwork. The pupil's own written questions build self-esteem and can develop independence. Some students will often come in with new related problems having investigated an area in more depth (self-initiated). Behind any task I want students to "reflect on and communicate about mathematics" (Carpenter,T., Fennema,E., Fuson, K., Hiebert, J., Human, P., Murray,H. et al. 1997 p. 30)

For me assessment is firstly a way for pupils, and me, to move forward. It is used to inform teaching; Do I need to teach a topic? Is the class is ready to move on? Was this method of teaching successful, or should it be adapted next time? It is all about the pupils (and I) being able to answer: Where am I?, Where am I going? and How do I move on?, (Black and Wiliam, 2009, p. 6) both as a teacher and with a particular class. I broadly agree with the assessment for learning factors mentioned by Broadfoot, Weeden and Winter, (2002 p. 24) although I do feel that very detailed specific learning goals could potentially take away some of the delight a new piece of maths can bring. When specifying learning goals I feel that an engaging and profitable way is to give an exemplar question and explain that by the end of the lesson pupils will have the tools, knowledge and understanding to attempt a solution to the problem.

The assessment I, and pupils, carry out is integral to everyday teaching. It takes a wide range of forms and is also influenced strongly by my constructive and social learning views. Dialogue and questioning happens on a day to day basis and this guides the course of a lesson, series of lessons and resources used. By asking students to clearly explain their thinking (to the class, each other or to me) we can gain an insight into where knowledge falls down or could be extended.

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Misconceptions are challenged head on and I will often use pivot questions to decide whether the next phase of a lesson can progress. These "questions that give us this "window into thinking"" (Wiliam, D. 1999,p. 16) can direct the phases of my lessons.

The quality of a students' invented question (with full mark schemes) is a good indicator as to their level of understanding. I believe that to write your own challenging question on a topic requires a much deeper level of understanding than answering one. This view is voiced by Wiliam (1999, p. 18): "There is substantial evidence that students' learning is enhanced by getting them to generate their own questions."

National exams for students are becoming higher stakes each year which does "transfer control over the curriculum to the agency that sets or controls exams" (Broadfoot, Weeden, Winter, 2002 p. 34). I realise the necessity for students to experience test conditions and exam style questions; this is one reason why I actually believe pencil and paper tests do have their place in lower school classes too, at this point in time they are low stakes and pupils can get rid of any uneasiness they feel about exams. As a department we have established unit tests. These are written by a maths teacher, sat in silence, marked by the teacher or peers and discussed in class. The results are logged and used a part of the evidence towards changing set and reporting attainment grades. I will sometimes exchange these teacher written tests for tests designed by students in a way to value their contribution (these are also sometimes exchanged with other classes) or a class may actually work through a test as a group activity (often with ready access to a detailed mark scheme). Most importantly revision resources are available (all year round) matching up with criteria for success, tests are always discussed and problem areas clarified (with groups and individuals), pupils have time to reflect on their own performance and all tests are kept by the pupil to aid further study. Each test ends with a more challenging application of the ideas met during the previous teaching; often proof related. Thus I believe that even these summative tests are used in an extremely formative way.

Curriculum Development

My aim was to produce a section of curriculum in which students would gain an understanding of Pythagoras'

formula, see and experience practical uses for it and extend their problem solving skills. The section of curriculum has been split into four sections: derivation of the formula, written group task, practical activity, and assessment.
Rather than simply give students the formula I have developed a task building on upon current knowledge. Exact learning objectives are not spelled out but students start by estimating part of the diagonal length of the room for a new projector cable and it is made clear that by the end of the lesson they will have a strategy to calculate the exact length necessary (it is assumed for the purpose of these resources that no pupil has full understanding of Pythagoras' theorem – if the class did contain such a pupil (s) they would act as a teaching aid in many of the tasks).
Students are given a pack of squares and asked to form triangles using the sides of these squares. Whilst doing this they are asked to write down the dimensions of the triangles, areas of the squares and geometric properties of the triangles. Ideas are recorded on a large sheet split according to type of triangle. This helps students to build on the models they have already formed regarding triangles, along the constructivist ideals. Students are encouraged to pattern spot. Furthermore they are asked to predict what type of triangle a particular set of squares would produce.

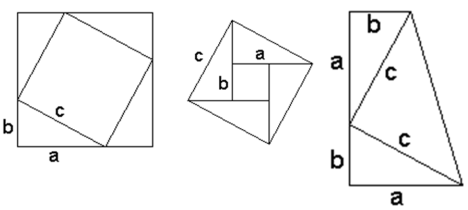
> **"There is substantial evidence that students' learning is enhanced by getting them to generate their own questions." (Williams, 1999)**

Students produce their own hypotheses and more able students are encouraged to record their ideas using their own symbols. Referred to as objectism in Bishop (1993 p. 103), this gives students a chance to develop their own symbols and to compare them to the established representations.
Eventually ideas are gathered together on the board and formalised, with notation and labelling clarified. Students are encouraged to think of an explanation as to why the sum of the two squares is equal to the square on the hypotenuse. At this point ideally the idea of proof should be mentioned and the validity of the statement "We have shown that Pythagoras' formula works for several right-angled triangles, therefore it is true for all right-angled triangles" be discussed. This would be a good example of rich questioning (William, 1999).
The homework task encourages pupils to consider the proof independently, return ready to share and then present the best ideas. Note that students don't have to use one of the given diagrams and they are welcome to come with an alternative method; all ideas will be valued and considered, their merits and pitfalls discussed; referred to as rationalism and control in Bishop (1993 p. 103).

Homework task

Pythagoras Proof

Use one of these diagrams to prove Pythagoras' Theorem



Before pupils leave they re-visit the starting question and in groups apply their knowledge in this practical context. Students present ideas and assess their own success against the solutions given, this results in the completion of an understanding scale:
In terms of being able to use Pythagoras' theorem in context in a right-angled triangle problem I am:

Utterly lost	Iffy	Completely happy & ready for more

Written group Task
Following a discussion and presentation (by pupils) of ideas on the proof of Pythagoras' theorem a group task is completed.
Each group of students is given a pack of cards. Each card contains a clue or is blank. No more information or direction is given to the students other than everything they need to know is on the cards. They are also informed that a blank card may be exchanged for the answer to one question from the teacher. However once all blank cards have been used no more questions can be asked of the teacher.
The clues themselves are all self-referencing. One card gives the initial task. If students need additional worksheets a card explains this. However to get any additional resources students always have to do a task first (given on the card). Amongst the cards is one explaining that any answers taken to the teacher must be written out in full. Once students have solved the initial task - finding the dimensions of shapes - they have to answer other questions about the individual and combined shapes. These final questions/tasks contain an extension and impossibility.
These resources reflect my beliefs about mathematics quite clearly. I want pupils to enjoy learning. Everyone enjoys solving puzzles; hence the pupils enjoy problem solving. By throwing away the text book pupils are more engaged – they feel like it is a game not a normal lesson. Each group will experience success whether it be calculating some diagonals or reaching the final few questions on the worksheet.
The task encourages pupils' to develop their own problem solving strategies. I don't advise students to keep a record of their working, to put used clues aside, to prioritise what questions they ask in return for their bank cards or to read all the information before they start. However, as pupils try the same task set up they start to develop these ideas for themselves. Pupils soon realise, when they have run up to shout answers to get the initial sheet, and are faced with a blank face and the comment to go and read your cards, that they need to read and absorb the information and instructions given. As pupils get to the worksheet, questions are designed such that the tasks are self-checking: if pupils haven't kept a clear record of their progress and ideas backtracking to find an error is very difficult so they may even have to start again; but they keep a record the next time!
Assessment of the task is extremely easy. All the pupils in the class are absorbed in the problem hence the teacher can circulate, listen in to conversations and question groups about their progress. Time can also be spent interviewing individuals or small groups. The teacher can assess the level of questions he/she is being asked by students in return for their blank cards – is the same clue card causing a problem for everyone? Information can be shared with small groups without solving the complete problem thus giving students the chance to continue reflecting "on the

solution and develop solution methods that they understand" (Carpenter,T., Fennema, E., Fuson, K., Hiebert, J., Human, P., Murray, H. et al. 1997 p. 36).

Pupils are challenged by the setting of a large problem but also in the way questions are worded. Topics are brought together and extended into more complicated problems. Worded questions can easily be incorporated into the clues. Furthermore towards the end of the worksheet impossibilities are considered and again the idea of justification through the use of proof; thus highlighting the rigour of the subject.

"Marking" of the diagrams sheet doesn't happen. Instead students check answers on the worksheet with the teacher, who simply identifies how many questions, if any, are incorrect (but initially not which of the answers is incorrect).

Differentiation of the task can be achieved in several ways:

- giving groups different numbers of help cards
- giving some groups hint cards or partial solutions on the first worksheet

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- giving some group a watered down version where some information is withheld so as not to confuse the group, but added as necessary.

Furthermore most of these levels of support can be added during the task if and as when needed.

Practical Activity

This section is to further enhance students understanding of the theorem but also to introduce some more specific uses of Pythagoras' theorem in a practical situation. A series of problems have been assembled with practical equipment some of which require the use of Pythagoras' theorem (but not all !)

In pairs or groups of three, students attempt to produce a written solution to any of the graded problems set up in the hall (pupils pick their own level). After some time groups swap answers and mark the solutions following a mark scheme. Alternative solutions are discussed between groups and as a whole class where the groups believe a 'better' solution has been found. Again this aligns itself closely with my social learning views that discussion aids learning and all views are considered and measured for their mathematical validity.

Assessment has been part of every stage of this process so far: questioning, self-assessment, and peer assessment.

Pythagoras' Theorem

Design your own question involving Pythagoras' theorem.

- Use a double clean page
- On the LHS write your question, including any diagrams
- On the RHS show a fully worked solution
- Remember your question should be challenging, include;
  - several steps
  - a bit of imagination
  - a worded problem
- Use the Princess Priscilla example to help you!

To encourage deeper understanding and a firmer grasp of the concept of Pythagoras' theorem students are asked to construct two questions. These are accompanied with fully worked solutions with marks. Students are given the associated part of the programme of study concerning Pythagoras.

The questions are photocopied (including the students name) and exchanged between members of the class. The questions received are attempted and then returned to the writer, who marks the answer given. The pair of students discuss who is right and who is wrong. This process is repeated several times over. At the end of this activity a class discussion ensues discussing the highlights of certain questions and points for improvements of others.

At this points students pick one of their questions with any improvements made and submit them as a final assessment piece. This is marked according to the criteria originally given. It acts as a final indication, which the teacher can record, of the level to which, and different situations in which, individuals can apply Pythagoras' theorem. The teacher can then identify a question from a text book that student should attempt to extend their understanding.

Final Thoughts

I feel that this series of tasks strongly represent my constructive and social views about learning. It also allows for a range of strong assessment techniques; each informs the pupil and teacher about progress, future ideas and improvements to be made. Students also experience a feeling of success through their problem solving, importance through the use of their own questions as an exercise, and challenge through the problem solving and practical tasks. They can also see the relevance of learning the theorem through the range of questions asked, particularly in the practical session.

The series of tasks contains a complete coverage of Pythagoras' theorem that goes beyond that of a basic text book set of exercises. Furthermore it improves problem solving skills, group work and engagement with mathematics. And student feedback has supported these assertions !

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